

Appl. No. 09/853,913
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 Reply to Office Action of June 13, 2006

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Amendments to the Claims:

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

- 1 Claims 1.-9. (canceled).
- 1 Claim 10. (currently amended) A method performed by a custodian computing
- 2 system, having memory, to share a secret S among n secret owners such that any k of the n secre:
- 3 owners may reconstruct the secret S , the method comprising the steps of:
- 4 choosing two large primes P and Q , such that PQ is greater than S when S is a
- 5 number;
- 6 storing in the custodian computer system memory a product $N = PQ$;
- 7 storing a product $M = (P-1)(Q-1)$;
- 8 choosing n random numbers e_1 through e_n that are relatively prime to N ;
- 9 choosing another random number e that is relatively prime to N ;
- 10 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for $1 \leq i \leq n$;
- 11 choosing another number d such that $ed \bmod M$ is equal to one;
- 12 storing a database of $\binom{n}{k}$ entries, wherein each entry is associated with a unique
- 13 combination of the $\binom{n}{k}$ possible combinations of the k secret owners, and wherein a particular
- 14 entry includes a value, c , that is the product of modulus M of d and the d_i values for i indices tha
- 15 correspond to the particular secret owners present in the unique combination for that particular
- 16 entry, wherein c corresponds to modulus M of the product ~~kd_i~~ kd_i ;
- 17 computing S^e ;
- 18 deleting from the custodian computer memory P , Q , and M ;
- 19 distributing n secret owner pieces to each of the n secret owners, wherein each of
- 20 the secret owner pieces includes S^e and one of the numbers e_1 through e_n ;

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21 deleting the secret S and e_1 through e_n , d_1 through d_n , and d ;
 22 receiving k secret owner values from a unique combination of k secret owners;
 23 determining a the value c that is associated with the unique combination; and
 24 determining the secret S using the value c retrieved from the database entry
 25 corresponding to the k secret owners whose secret owner pieces have been received and $S^c \bmod$
 26 N .

1 Claim 11. (previously presented) A method as in claim 10, wherein receiving k
 2 secret owner values from the unique combination of k secret owners comprises:
 3 receiving a first of the n secret owner pieces from one of the n secret owners; and
 4 computing and storing $S' = S^{e_f} \bmod N$, where f represents the one of the numbers
 5 e_1 through e_n contained in the first of the n secret owner pieces.

1 Claim 12. (previously presented) A method as in claim 11, wherein receiving k
 2 secret owner values from the unique combination of k secret owners comprises:
 3 receiving a second of the n secret owner pieces from another one of the n secret
 4 owners;
 5 computing $S^q \bmod N$, where q represents the one of the numbers e_1 through e_n
 6 contained in the second of the n secret owner pieces; and replacing S' with $S^q \bmod N$.

1 Claim 13. (previously presented) A method as in claim 12, wherein receiving k
 2 secret owner values from the unique combination of k secret owners comprises:
 3 each time another of the secret owner pieces is received from another one of the
 4 secret owners;
 5 computing $S^q \bmod N$, where q represents the one of the numbers e_1 through e_n
 6 contained in the another of the n secret owner pieces; and replacing S' with $S^q \bmod N$.

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1 Claim 14. (previously presented) A method as in claim 13, further comprising
 2 the steps of:
 3 after k secret owner pieces have been received,
 4 retrieving from the database the value c from among the $\binom{n}{k}$ values, wherein the
 5 value c corresponds to the k secret owner pieces of the unique combination of k secret owners
 6 that were received by the custodian;
 7 computing $S^c \bmod N$; and
 8 replacing S' with $S^c \bmod N$.

1 Claim 15. (currently amended) A method performed by a custodian computing
 2 system, having memory, to share a secret S among n secret owners such that any k of the n secre
 3 owners may reconstruct the secret, the method comprising the steps of:
 4 choosing two large primes P and Q , such that PQ is greater than S where S is a
 5 number;
 6 storing in the custodian computer memory a product $N = PQ$;
 7 storing a product $M = (P-1)(Q-1)$;
 8 choosing n random numbers e_1 through e_n that are relatively prime to N ;
 9 choosing random numbers e and e' that are relatively prime to N ;
 10 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for $1 \leq i \leq n$;
 11 choosing numbers d and d' such that $ed \bmod M$ is equal to one and such that $e'd'$
 12 $\bmod M$ is equal to one;
 13 storing a database of $\binom{n}{k}$ entries, wherein each entry is associated with a unique
 14 combination of the $\binom{n}{k}$ possible combinations of the k secret owners, and wherein a particular
 15 entry includes a value, c , that is the product of modulus M of d and the d_i values for i indices tha

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16 correspond to the particular secret owners present in the unique combination for that particular
 17 entry, wherein c corresponds to modulus M of the product ~~$k d_i$~~ $k d_i$;
 18 computing $S^{ee'}$;
 19 deleting from the custodian computer memory P , Q , and M ;
 20 distributing n secret owner pieces to each of the n secret owners, wherein each of
 21 the secret owner pieces includes $S^{ee'}$ and one of the numbers e_1 through e_n ;
 22 deleting the secret S and e_1 through e_n , d_1 through d_n , and d
 23 receiving k secret owner values from a unique combination of k secret owners;
 24 retrieving from the database the value c that is associated with the unique
 25 combination; and
 26 determining the secret S using the value c and the k secret owner value.

1 Claim 16. (previously presented) A method as in claim 15, wherein receiving k
 2 secret owner values from the unique combination of k secret owners comprises:
 3 receiving a first of the n secret owner pieces from one of the n secret owners; and
 4 computing and storing $S' = S^{ef} \bmod N$, where f represents the one of the numbers
 5 e_1 through e_n contained in the first of the n secret owner pieces.

1 Claim 17. (previously presented) A method as in claim 16, wherein receiving k
 2 secret owner values from the unique combination of k secret owners comprises:
 3 receiving a second of the n secret owner pieces from another one of the n secret
 4 owners;
 5 computing $S^q \bmod N$, where q represents the one of the numbers e_1 through e_n
 6 contained in the second of the n secret owner pieces; and replacing S' with $S^q \bmod N$.

1 Claim 18. (previously presented) A method as in claim 17, wherein receiving k
 2 secret owner values from the unique combination of k secret owners comprises:
 3 each time another of the secret owner pieces is received from another one of the
 4 secret owners;

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5 computing $S^q \bmod N$, where q represents the one of the numbers e_1 through e_n
 6 contained in the another of the n secret owner pieces; and replacing S' with $S^q \bmod N$.

1 Claim 19. (previously presented) A method as in claim 18, further comprising
 2 the steps of:

3 after k secret owner pieces have been received,

4 retrieving from the database the value c from among the $\binom{n}{k}$ values, wherein the

5 value c corresponds to the k secret owner pieces from the unique combination of k secret owner
 6 that were received by the custodian;

7 computing $S^c \bmod N$;

8 replacing S' with $S^c \bmod N$;

9 computing $S^{d'} \bmod N$; and

10 replacing S' with $S^{d'} \bmod N$.

1 Claim 20. (currently amended) A method performed by a custodian computing
 2 system, having memory, to share a secret among n secret owners such that any k of the n secret
 3 owners may reconstruct the secret, the method comprising the steps of:

4 encrypting the secret so as to generate an encrypted secret;

5 deleting from the custodian computer memory the secret; and

6 performing a forward k out of n secret sharing algorithm on the encrypted secret
 7 so as to generate n secret owner pieces;

8 storing in a database a plurality of entries associated with a plurality of unique
 9 combinations of k secret owners of the n secret owners, wherein a particular entry includes a

10 value, c , that is the a product of modulus M of d and the d_i values for i indices that correspond to
 11 the particular secret owners present in the unique combination for that particular entry, wherein
 12 corresponds to modulus M of the product $k d_i$ $k d_i$;

13 distributing the n secret owner pieces to the n secret owners;

14 receiving k secret owner values from a unique combination of k secret owners;

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15 retrieving from the database a value c that is associated with the unique
16 combination;
17 performing a reverse k out of n secret sharing algorithm on the k secret owner
18 pieces so as to recreate the encrypted secret using the value c ; and
19 decrypting the encrypted secret so as to recreate the secret.

Claims 21. - 24. (canceled).

1 Claim 25. (original) A method as in claim 20, wherein the step of performing a
2 forward k out of n secret sharing algorithm includes the steps of:
3 dividing the encrypted secret into k pieces; and
4 performing n polynomial evaluations at n points of a degree- k polynomial using
5 the k pieces of the encrypted secret as polynomial coefficients;
6 wherein each of the k secret owner pieces includes a result of one of the n
7 polynomial evaluations and a corresponding one of the n points.

1 Claim 26. (previously presented) A method as in claim 25, wherein the step of
2 performing a reverse k out of n secret sharing algorithm includes the steps of generating a system
3 of k linear equations and solving the system of k linear equations for the k pieces of the encrypted
4 secret.

1 Claim 27. (previously presented) A method as in claim 26, further comprising
2 the step of:
3 assembling the k pieces of the encrypted secret so as to recreate the encrypted
4 secret.

Claims 28.-29. (canceled).

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Claim 30. (currently amended) A computer readable storage medium having embodied thereon computer readable program code suitable for programming a computer to perform a method performed by a custodian to share a secret S among n secret owners such that any k of the n secret owners may reconstruct the secret, the method comprising the steps of:

- choosing two large primes P and Q , such that PQ is greater than S where S is a number;
- computing and storing a product $N = PQ$;
- computing and storing a product $M = (P-1)(Q-1)$;
- choosing n random numbers e_1 through e_n that are relatively prime to N ;
- choosing another random number e that is relatively prime to N ;
- choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for $1 \leq i \leq n$;
- choosing another number d such that $ed \bmod M$ is equal to one;
- storing a database of $\binom{n}{k}$ entries, wherein each entry is associated with a unique combination of the $\binom{n}{k}$ possible combinations of the k secret owners, and wherein a particular entry includes a value, c , that is the product of modulus M of d and the d_i values for i indices that correspond to the particular secret owners present in the unique combination for that particular entry, wherein c corresponds to modulus M of the product $\prod d_i$;
- computing S^e ;
- deleting P , Q , and M ;
- distributing n secret owner pieces to each of the n secret owners, wherein each of the secret owner pieces includes S^e and one of the numbers e_1 through e_n ;
- deleting the secret S and e_1 through e_n , e , d_1 through d_n , and d ;
- receiving k secret owner values from a unique combination of k secret owners;
- retrieving from the database one of the values c that is associated with the unique combination; and
- determining the secret S using the value c and the k secret owner values.

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Claim 31. (currently amended) A computer readable storage medium having embodied thereon computer readable program code suitable for programming a computer to perform a method performed by a custodian to share a secret S among n secret owners such that any k of the n secret owners may reconstruct the secret, the method comprising the steps of:

- choosing two large primes P and Q , such that PQ is greater than S where S is a number;
- storing a product $N = PQ$;
- storing a product $M = (P-1)(Q-1)$;
- choosing n random numbers e_1 through e_n that are relatively prime to N ;
- choosing random numbers e and e' that are relatively prime to N ;
- choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for $1 \leq i \leq n$;
- choosing numbers d and d' such that $ed \bmod M$ is equal to one and such that $e'd' \bmod M$ is equal to one;
- generating and storing a database of $\binom{n}{k}$ values, where each value is the product of d and a unique k of the d_i numbers for $1 \leq i \leq n$, wherein each value is associated with a unique combination of k secret owners of the n secret owners;
- storing a database of $\binom{n}{k}$ entries, wherein each entry is associated with a unique combination of the $\binom{n}{k}$ possible combinations of the k secret owners, and wherein a particular entry includes a value, c , that is the product of modulus M of d and the d_i values for i indices that correspond to the particular secret owners present in the unique combination for that particular entry, wherein c corresponds to modulus M of the product $\cancel{k d_i} \ k d_i$;
- computing $S^{ee'}$;
- deleting P , Q , and M ;

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24 distributing n secret owner pieces to each of the n secret owners, wherein each of
 25 the secret owner pieces includes $S^{ee'}$ and one of the numbers e_1 through e_n ;
 26 deleting the secret S and e_1 through e_n , d_1 through d_n , and d ;
 27 receiving k secret owner values from a unique combination of k secret owners;
 28 retrieving from the database one of the values c that is associated with the unique
 29 combination; and
 30 determining the secret S using the value c and the k secret owner values.

1 Claim 32. (previously presented) A computer readable storage medium having
 2 embodied thereon computer readable program code suitable for programming a computer to
 3 perform a method performed by a custodian to share a secret among n secret owners such that
 4 any k of the n secret owners may reconstruct the secret, the method comprising the steps of:
 5 encrypting the secret so as to generate an encrypted secret;
 6 deleting the secret;
 7 performing a forward k out of n secret sharing algorithm on the encrypted secret
 8 so as to generate n secret owner pieces;
 9 storing in a database a plurality of entries associated with a plurality of unique
 10 combinations of k secret owners of the n secret owners, wherein a particular entry includes a
 11 value, c , that is the a product of modulus M of d and ~~the~~ d_i values for i indices that correspond to
 12 the particular secret owners present in the unique combination for that particular entry, wherein d
 13 corresponds to modulus M of the product ~~$k d_i$~~ $k d_i$;
 14 distributing the n secret owner pieces to the n secret owners;
 15 receiving k secret owner values from a unique combination of k secret owners;
 16 retrieving from the database one of the values c that is associated with the unique
 17 combination;
 18 performing a reverse k out of n secret sharing algorithm on the k secret owner
 19 pieces so as to recreate the encrypted secret using the value c ; and
 20 decrypting the encrypted secret so as to recreate the secret.

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Claims 33.-34. (canceled).

1 Claim 35. (currently amended) A computer comprising a processor and a
 2 computer readable storage medium coupled to the processor having embodied thereon processo
 3 readable program code suitable for programming a computer to perform a method performed by
 4 a custodian to share a secret S among n secret owners such that any k of the n secret owners may
 5 reconstruct the secret, the method comprising the steps of:
 6 choosing two large primes P and Q , such that PQ is greater than S where S is a
 7 number;
 8 storing a product $N = PQ$;
 9 storing a product $M = (P-1)(Q-1)$;
 10 choosing n random numbers e_1 through e_n that are relatively prime to N ;
 11 choosing another random number e that is relatively prime to N ;
 12 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for $1 \leq i \leq n$;
 13 choosing another number d such that $ed \bmod M$ is equal to one;
 14 storing a database of $\binom{n}{k}$ entries, wherein each entry is associated with a unique
 15 combination of the $\binom{n}{k}$ possible combinations of the k secret owners, and wherein a particular
 16 entry includes a value, c , that is the product of modulus M of d and the d_i values for i indices tha
 17 correspond to the particular secret owners present in the unique combination for that particular
 18 entry, wherein c corresponds to modulus M of the product $k d_i$ kd_i ;
 19 computing S^e ;
 20 deleting P , Q , and M ;
 21 distributing n secret owner pieces to each of the n secret owners, wherein each of
 22 the secret owner pieces includes S^e and one of the numbers e_1 through e_n ;
 23 deleting the secret S and e_1 through e_n , d_1 through d_n , and d ;
 24 receiving k secret owner values from a unique combination of k secret owners;

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25 retrieving from the database one of the a values c that is associated with the
 26 unique combination; and
 27 determining the secret S using the value c and the k secret owner values.

1 Claim 36. (currently amended) A computer comprising a processor and a
 2 computer readable storage medium coupled to the processor having embodied thereon processor
 3 readable program code suitable for programming the computer to perform a method performed
 4 by a custodian to share a secret S among n secret owners such that any k of the n secret owners
 5 may reconstruct the secret, the method comprising the steps of:

6 choosing two large primes P and Q , such that PQ is greater than S where S is a
 7 number;

8 storing a product $N = PQ$;

9 storing a product $M = (P-1)(Q-1)$;

10 choosing n random numbers e_1 through e_n that are relatively prime to N ;

11 choosing random numbers e and e' that are relatively prime to N ;

12 choosing n numbers d_1 through d_n such that $e_i d_i \bmod M$ equals one for $1 \leq i \leq n$;

13 choosing numbers d and d' such that $ed \bmod M$ is equal to one and such that $e'd'$
 14 $\bmod M$ is equal to one;

15 generating and storing a database of $\binom{n}{k}$ values, where each value is the product
 16 of d and a unique k of the d_i numbers for $1 \leq i \leq n$, wherein each value is associated with a
 17 unique combination of k secret owners of the n secret owners;

18 storing a database of $\binom{n}{k}$ entries, wherein each entry is associated with a unique

19 combination of the $\binom{n}{k}$ possible combinations of the k secret owners, and wherein a particular
 20 entry includes a value, c , that is the product of modulus M of d and the d_i values for i indices that
 21 correspond to the particular secret owners present in the unique combination for that particular
 22 entry, wherein c corresponds to modulus M of the product $\prod_{i \in k} d_i$;

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23 computing $S^{ee'}$;
 24 deleting P , Q , and M ;
 25 distributing n secret owner pieces to each of the n secret owners, wherein each of
 26 the secret owner pieces includes $S^{ee'}$ and one of the numbers e_1 through e_n ;
 27 deleting the secret S and e_1 through e_n , d_1 through d_n , and d ;
 28 receiving k secret owner values from a unique combination of k secret owners;
 29 retrieving from the database one of the values c that is associated with the unique
 30 combination; and
 31 determining the secret S using the value c and the k secret owner values.

1 Claim 37. (currently amended) A computer comprising a processor and a
 2 computer readable storage medium coupled to the processor having embodied thereon processor
 3 readable program code suitable for programming the computer to perform a method performed
 4 by a custodian to share a secret among n secret owner such that any k of the n secret owners may
 5 reconstruct the secret, the method comprising the steps of:
 6 encrypting the secret so as to generate an encrypted secret;
 7 deleting the secret;
 8 performing a forward k out of n secret sharing algorithm on the encrypted secret
 9 so as to generate n secret owner pieces;
 10 storing in a database a plurality of entries associated with a plurality of unique
 11 combinations of k secret owners of the n secret owners, wherein a particular entry includes a
 12 value, c , that is the a product of modulus M of d and the d_i values for i indices that correspond to
 13 the particular secret owners present in the unique combination for that particular entry, wherein
 14 corresponds to modulus M of the product ~~$k d_i$~~ $k d_i$;
 15 distributing the n secret owner pieces to the n secret owners;
 16 receiving k secret owner values from a unique combination of k secret owners;
 17 retrieving from the database one of the values c that is associated with the unique
 18 combination;

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- 19 performing a reverse k out of n secret sharing algorithm on the k secret owner
- 20 pieces so as to recreate the encrypted secret using the value c ; and
- 21 decrypting the encrypted secret so as to recreate the secret.